

## **Inverse Kinematics of a Hyper-Redundant Robotic Manipulator**

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### **ABSTRACT**

Serial robots, Snake or, Worm robots, Tentacle robots or continuum robots are known as hyper-redundant robots and possess very large degrees of kinematics redundancy. Inverse kinematics of hyper-redundant robots can have infinite number of solutions, which is a great challenge against position control of such robots. Various techniques have so far been proposed for inverse kinematics of hyper-redundant robots that involve wide range of mathematics, which include nonlinear optimization, Artificial Neural Network, Fuzzy System etc. In this paper a new technique has been proposed that assumes a configuration with a virtual layer, where probable singularities are included. In the successive steps the singularities are removed following some geometric propositions to have the final version of the configuration, which ultimately gives the inverse kinematics of the hyper-redundant robot. Mathematics involved in this new technique is the traditional inverse kinematics solution of two link subrobots, which are selected to satisfy the geometric propositions. The proposed technique has been tested on four link and six link robots and some comparison are made with one of the recent techniques known as ANFIS.

### **1. INTRODUCTION**

Hyper-redundant robots have lots of advantages over the nonredundant robots for the flexibility of configurations in achieving the same goal. Due to the flexibility of configurations such robots are able to continue operations successfully even after mechanical failure of few joints. However, it is not an easy task to manage this flexibility due to the involvement of mathematical complexity in determining inverse kinematics (IKS) of these robots, especially, handling of pseudo inverse of non-square matrices. Literature on hyper-redundant robots shows various approaches for their inverse kinematic solutions. Chirikjian and Burdick [1,2] approximated continuous curve through the joints of the robots using Bessel function and then applied differential geometry, which helps avoid calculation of pseudo inverse matrices. This method works well for high level of redundancy that occurs in continuous robots and shows poor performance for reduced level of redundancy in discrete robots. Variable length links and artificial potential functions are used in [3] in reaching desired positions avoiding obstacles. References [4,5] applied optimization techniques in solving IKS of hyper-redundant robots, while [6-8] used ANFIS, Fuzzy logic or ANN. All these methods discussed involve complex mathematics, and some of them take considerable time to come up with useful IKS solution. When a manipulator is made of more than three links, the conventional technique of finding the joint variables is not useful. To come up with inverse kinematic solution the hyper-redundant robots are divided into small sub-robots and the small sub-robots are controlled separately and successively to control the parent robot [7].

Asl et al [9] came up with two IKS techniques of hyper-redundant robots, where one of these techniques known as VLA assumes few virtual layers and the other one known as DDS uses proportional distribution of displacements of the virtual layers. The DDS is used when the VLA gives singular configuration. In the present paper we have proposed a similar method with virtual layers. However, this new method instead of checking singular configuration at every step assumes the first virtual layer in singular configuration and then gradually removes the singularities to arrive at the final configuration that gives a coil shape.

## 2. ALGORITHM FOR NEW TECHNIQUE

The proposed algorithm, at the beginning, considers some virtual configuration with singularities. The virtual configuration subsequently is reconfigured to eliminate the singularities. Steps of the algorithm are discussed in conjunction with the Fig.1. Let us consider O-1-2-3-4 be the initial configuration of the robot consisting of four links and P be the desired point to be reached. Let us divide the hyper-redundant robot into two subrobots: subrobot 1 (SR1) consists of the first two links (L1 and L2) and subrobot 2 (SR2) consists of the last two links (L3 and L4). From the known lengths of the links the configuration O-A-P can easily be determined considering the SR1 and SR2 in their singular configurations respectively. This configuration along with the position vector OP forms the triangle OAP. In the next step we locate the centroid of the triangle OAP and determine inverse kinematics of the two subrobots in such a way that for SR1, C is the desired point; and for SR2, C is the base point and P is the desired point. In the case of links of almost equal lengths, the distances OC and CP will be shorter than that of the respective subrobots. This geometric feature will help remove the singularities from both the subrobots.

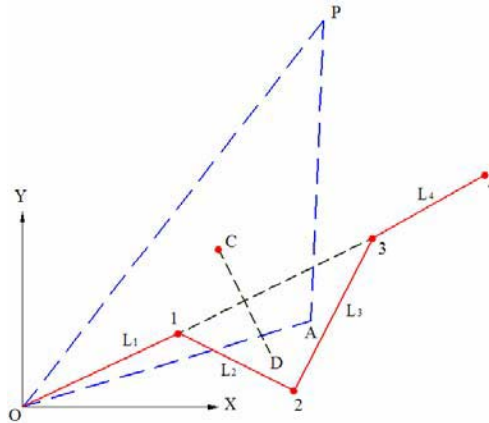


Fig.1 Virtual layers and the final configuration

In determining the subrobot configurations, if both the subrobots are configured elbow up, links L2 and L3 together may become elbow down. To make this portion elbow up we can simply reconfigure the links L2 and L3 shifting the point C symmetrically on the other side of the line joining the points 1' and 3', in Fig.1 say at D. This fine tuning will make the whole robot elbow up.

### 2.1 Kinematic Analysis of 2-Link subrobots

Fig.2 shows the traditional 2-link manipulator that has been considered as 2-link subrobot in the proposed technique.

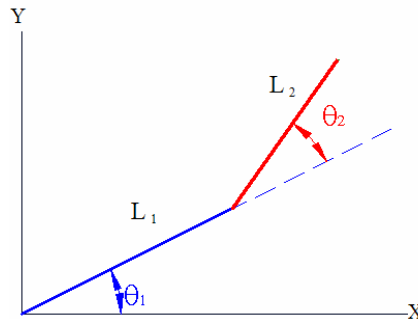


Fig.2 2-Link manipulator

Once two consecutive links (in the case of virtual layers two or more links in singular configuration is assumed as a single link) with their arm lengths are known ( say  $L_1$  and  $L_2$ ) the joint variables  $\theta_1$  and  $\theta_2$  for specified tip position (x,y) can be determined from the following forward kinematic equations.

$$x = l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) \quad (1)$$

$$y = l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2) \quad (2)$$

Solutions of these equations are available in any book on robotics and are presented in equation (3).

$$\theta_1 = a \tan 2(\sin \theta_1, \cos \theta_1) \quad (3)$$

$$\theta_2 = a \tan 2(\sin \theta_2, \cos \theta_2)$$

Where,

$$\cos \theta_2 = \frac{x_n^2 + y_n^2 - l_1^2 - l_2^2}{2l_1 l_2}; \quad \sin \theta_2 = \pm \sqrt{1 - (\cos \theta_2)^2}; \quad \sin \theta_1 = \frac{k_1 y_n - k_2 x_n}{k_1^2 + k_2^2};$$

$$\cos \theta_1 = \frac{k_1 x_n + k_2 y_n}{k_1^2 + k_2^2}; \quad k_1 = (l_1 + l_2 \cos \theta_2) \cos \theta_1; \quad \text{and, } k_2 = l_2 \sin \theta_2$$

### 3. INVERSE KINEMATICS FOR 4-LINK AND 6-LINK ROBOTS

#### 3.1 Inverse Kinematics for 4-Link Robot

The new algorithm has been applied for one 4-link and one 6-link robots respectively. In Fig.3 and Fig.4 IKS for 4-link robots have been presented for two different desired points, where one is close to the outer boundary of the workspace and the other close to the base of the robot respectively. In both the cases the IKS are solved for elbow up configuration so that the robot configuration can be shaped as a coil, a configuration often seen in arthropods like spiders or crabs. The joint variables and tracking error are plotted for five seconds in Figs. 5 and 6 respectively. The changes of Joint variables are smooth and the tracking errors are negligible ( $10^{-17}$  m).

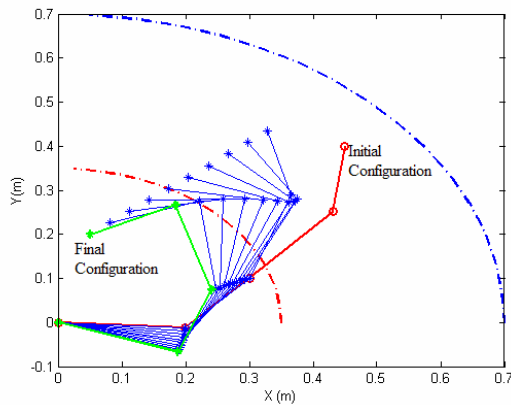


Fig. 3 IKS Configuration of a 4-Link robot for a point close to the base point

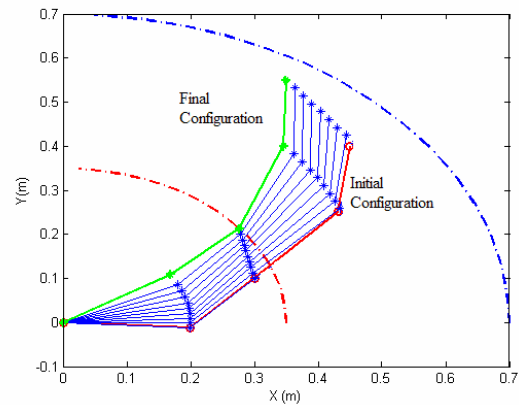


Fig. 4 IKS Configuration of a 4-Link robot for a point close to the boundary

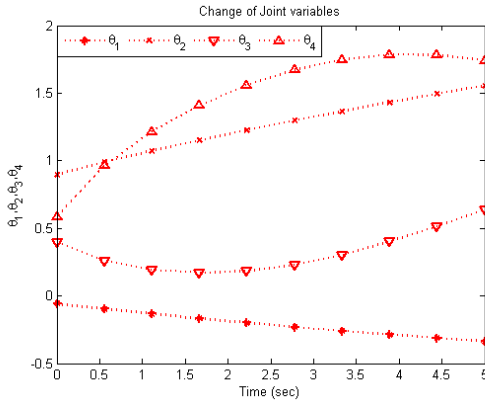


Fig. 5 Joint Values of 4-Link using IKS for inner point

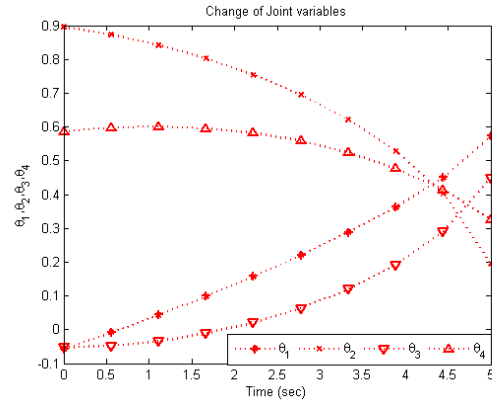


Fig. 6 Joint Values of 4-Link using IKS for outer point

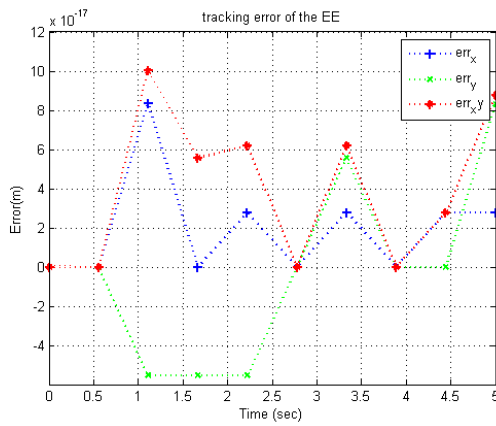


Fig. 7 Tracking error of 4-Link using IKS for Inner point

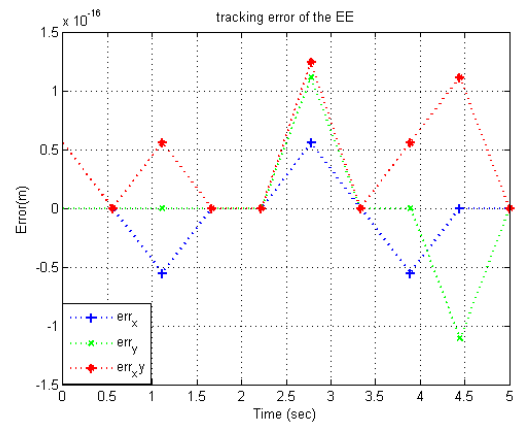


Fig. 8 Tracking error of 4-Link using IKS for outer point

### 3.2 Inverse Kinematics for 6-Link Robot

The same algorithm is then extended for 6-link robot with similar desired points. The elbow up configurations in the both the cases are shown in Figs. 9 and 10. As a result, the robot is finally configured in coil shape. Change of joint variables and tracking errors shown in Figs. 11 to 14 show similar trends like 4-link robots.

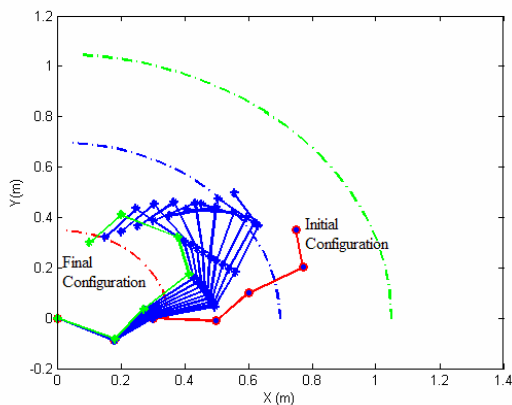


Fig. 9 IKS Configuration of a 6-Link robot for a point close to the base point

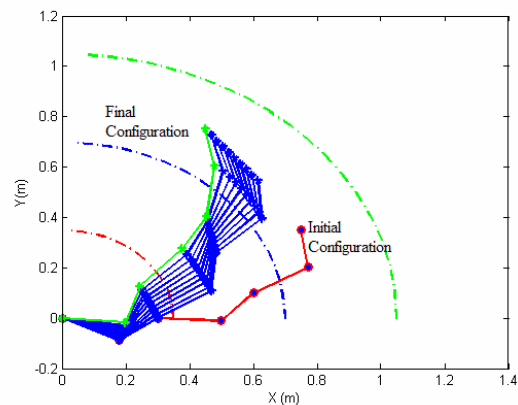


Fig. 10 IKS Configuration of a 6-Link robot for a point close to the boundary

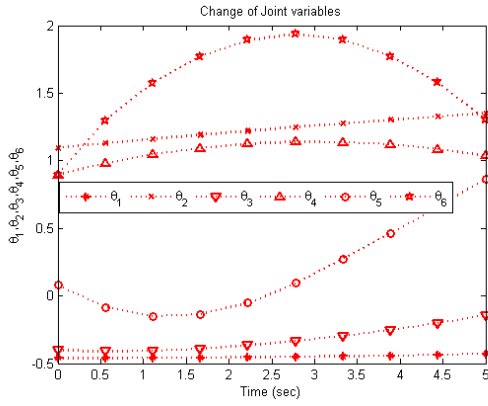


Fig. 11 Joint Values of 6-Link using IKS for outer point

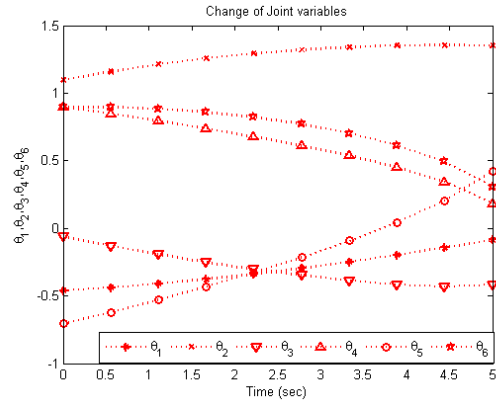


Fig. 12 Joint Values of 6-Link using IKS for outer point

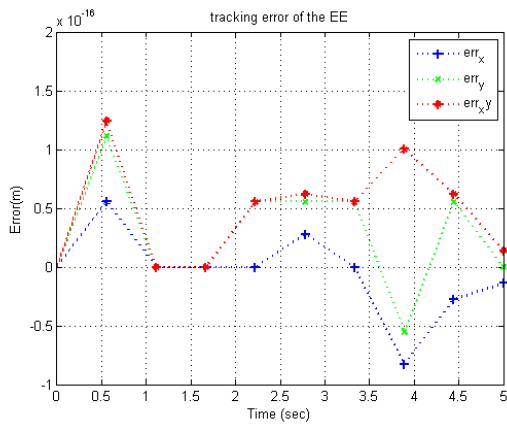


Fig. 13 Tracking error of 6-Link using IKS for Inner point

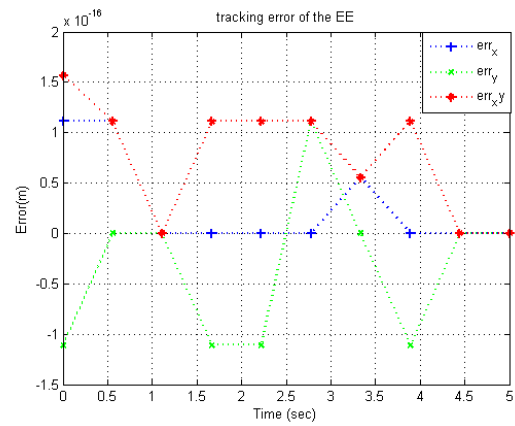


Fig. 14 Tracking error of 6-Link using IKS for Outer point

### 3.3 New method vs ANFIS

To compare the new method with the one done using ANFIS, the above 4-link and 6-link robots were run using ANFIS. The results of the 4-link and 6-link robots are shown in Fig. 15 and 16 respectively. The results are quite erratic. On the other hand training of 4-link and 6-link robots take quite a long time compared to that by the proposed method. Thus it shows the proposed method is simple, fast and gives singularity free solution.

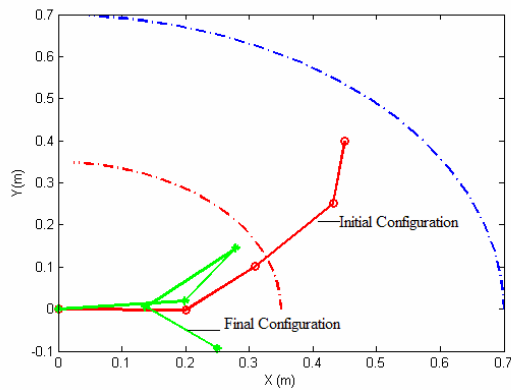


Fig. 15 Configuration of 4-Link using ANFIS

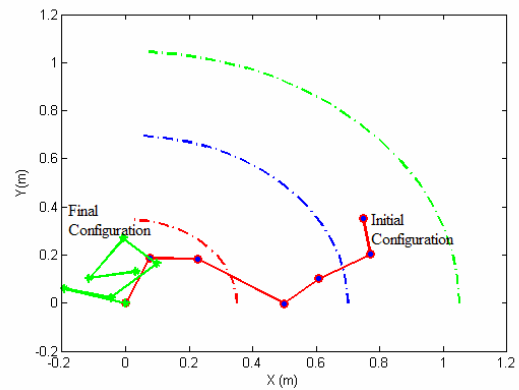


Fig. 16 Configuration of 6-Link using ANFIS

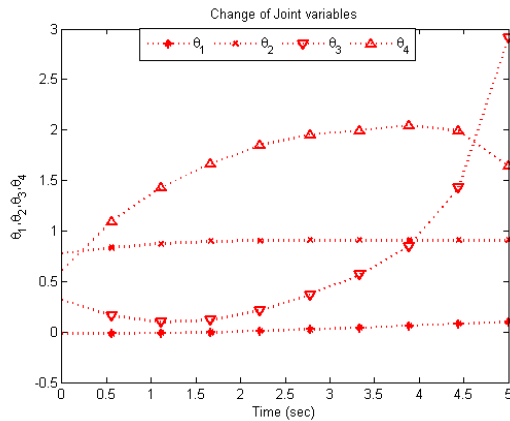


Fig.17 Joint Values of 4-Link using ANFIS

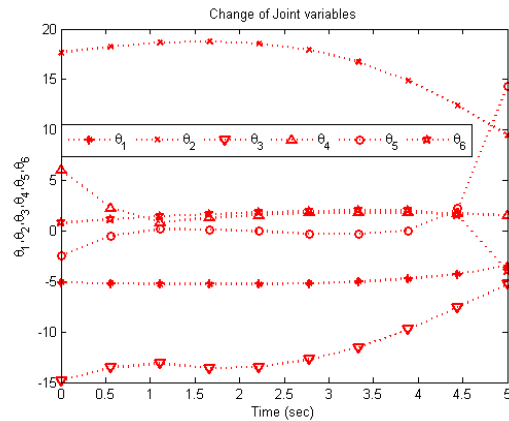


Fig.18 Joint Values of 6-Link using ANFIS

## CONCLUSIONS

A new inverse kinematics technique is developed for hyper-redundant robot which starts with a virtual layer and singular configuration. The singular configuration is then gradually removed using simple geometric propositions to achieve a coil shaped configuration at the final stage. Such coil shaped configuration helps avoid obstacles between the base of the robot and the desired point. Comparison of simulation results with that of the ANFIS show that the new method is simple and requires less computation than ANFIS.

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